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Proposition 2. *The conditions of theorem 2 are satisfied for the Kerner–Konäuser model (16) whenever d_1 is small an negative, and $d_2, d_3 > 0$.*

Proof. the condition $v_e''(v_c) = 0$ reduces to

$$e^{\frac{q_g}{d_3 x}}(q_g - 2d_3 x) - e^{d_2/d_3}(q_g + 2d_3 x) = 0, \tag{17}$$

and the nondegenericity condition (14) to

$$e^{\frac{q_g}{d_3 x}}(q_g - d_3 x) + e^{d_2/d_3}(q_g + d_3 x) = 0. \tag{18}$$

The change of variable $q_g = zx$ (recall $x = v + v_g$, thus $z = \rho/\rho_{max}$) and reduces each equation to the trivial one $x = 0$ which is discarded since $v \neq -v_g$, or

$$2d_3 e^{d_2/d_3} + (2d_3 - z)e^{z/d_3} + e^{d_2/d_3} z = 0 \tag{19}$$
$$d_3 e^{d_2/d_3} + (d_3 - z)e^{z/d_3} + e^{d_2/d_3} z = 0 \tag{20}$$

Taking the difference of these yields $d_3 e^{z/d_3} = -d_3 e^{d_2/d_3}$. Since all constants d_1, d_2, d_3 are positive, this last equation has no solution for $z > 0$. □

Corolary 1. *The Kerner–Konäuser model (3) with the fundamental diagram (16) under the assumption (1) has traveling wave solutions in the unbounded domain $x \in (-\infty, \infty)$ and in the bounded domain $[0, L]$ with periodic boundary conditions.*

Proof. From the BT Theorem 2, it follows that there exists Hopf limit cycles $(v(z), y(z))$, where $z = \rho_{max}\xi$ and $\xi = x + V_g t$ (see (5)). This yields a traveling wave solution of (3) in the form $V(x,t) = V_{max} v(x + V_g t)$. If T is commensurable with L then the traveling wave satisfies also periodic boundary conditions. □

References

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