3. Case II. As a further illustration of this method let us consider the integral of $g(x)$ where $g(x)=[F(x)]^pR(x)$, $F(x)$ and $R(x)$ being polynomials and p a non-integral fraction. We assume here that the integral has the form:

$$\int g(x)dx = (ax+b)[F(x)]^{p+1}+c \tag{6}$$

where a, b, and c are constants. Differentiating we obtain:

$$[F(x)]^pR(x) = [F(x)]^p\{aF(x)+(p+1)(ax+b)F'(x)\}. \tag{7}$$

Hence formula (6) works if we can find a and b such that:

$$R(x) = aF(x)+(p+1)(ax+b)F'(x). \tag{8}$$

We illustrate with a numerical example:

$$I=\int \frac{5x^3-3x^2+6x-3}{\sqrt{2x^3+4x-1}}\,dx.$$

Then we must find a and b such that:

$$\begin{aligned}5x^3-3x^2+6x-3 &= a[2x^3+4x-1]+\tfrac{1}{2}(ax+b)(6x^2+4)\\ &= 5ax^3+3bx^2+6ax+2b-a.\end{aligned}$$

This is consistent in a and b, giving $a=1$ and $b=-1$. Hence:

$$I=(x-1)\sqrt{2x^3+4x-1}+c.$$

SOME INTEGRAL FORMULAS

H. W. SMITH, Oklahoma A. and M. College

When we orthonormalize the set of harmonic functions $r^n \cos n\theta$, $r^n \sin n\theta$ over the square of side 2 with center at the origin and sides parallel to the coordinate axes, integrals of the type

$$\int \sec^k\theta\cos p\theta d\theta, \qquad \int \sec^k\theta\sin p\theta d\theta$$

$$\int \csc^k\theta\cos p\theta d\theta, \qquad \int \csc^k\theta\sin p\theta d\theta$$

with appropriate limits must be computed.

Formulas 369–372 of Pierce's tables may be used as reduction formulas for these integrals, but they are given there as the sum of two other integrals of the same form.

To obtain reduction formulas that involve only one integral in the right members for the first two of these we integrate

$$\int \sec^k \theta e^{ip\theta} d\theta$$

by parts, to obtain

$$\int \sec^k \theta e^{ip\theta} d\theta = 1/ip \sec^k \theta e^{ip\theta} - k/ip \int \sec^k \theta \tan \theta e^{ip\theta} d\theta. \tag{1}$$

Integrating by parts again we find

$$\int \sec^k \theta \tan \theta e^{ip\theta} d\theta = 1/ip \sec^k \theta \tan \theta e^{ip\theta} - \frac{k+1}{ip} \int \sec^{k+2} \theta e^{ip\theta} d\theta$$
$$+ k/ip \int \sec^k \theta e^{ip\theta} d\theta.$$

Substitute this result into the right member of (1), solve for $\int \sec^{k+2} \theta e^{ip\theta} d\theta$ and separate the real and imaginary parts to obtain:

$$\int \sec^{k+2} \theta \cos p\theta d\theta = \frac{\sec^k \theta}{k(k+1)} [k \tan \theta \cos p\theta + p \sin p\theta]$$
$$+ \frac{k^2 - p^2}{k(k+1)} \int \sec^k \theta \cos p\theta d\theta.$$
$$\int \sec^{k+2} \theta \sin p\theta d\theta = \frac{\sec^k \theta}{k(k+1)} [k \tan \theta \sin p\theta - p \cos p\theta]$$
$$+ \frac{k^2 - p^2}{k(k+1)} \int \sec^k \theta \sin p\theta d\theta.$$

The last two integrals can be found in the samé way and are:

$$\int \csc^{k+2} \theta \cos p\theta d\theta = \frac{-\csc^k \theta}{k(k+1)} [k \cot \theta \cos p\theta - p \sin p\theta]$$
$$+ \frac{k^2 - p^2}{k(k+1)} \int \csc^k \theta \cos p\theta d\theta.$$
$$\int \csc^{k+2} \theta \sin p\theta d\theta = \frac{-\csc^k \theta}{k(k+1)} [k \cot \theta \sin p\theta + p \cos p\theta]$$
$$+ \frac{k^2 - p^2}{k(k+1)} \int \csc^k \theta \sin p\theta d\theta.$$

The case appearing most frequently in the particular problem above is that in which $k = p = 2n$, so that in this case the coefficient of the integral on the right is zero.