$$\cos\theta = \frac{1-t^2}{1+t^2}.$$

The neatness of this method of solution will be shown by applying it to an example:

EXAMPLE. *Find the angles between* $-\pi$ *and* π *radians such that*

$$5\sin\theta - 21\cos\theta = 10.$$

On making the above substitutions one gets

$$\frac{10t}{1+t^2} - 21\,\frac{1-t^2}{1+t^2} = 10.$$

When this equation is cleared of fractions and terms collected the result is

$$11t^2 + 10t - 31 = 0.$$

Hence,

$$t = \frac{-10 \pm \sqrt{100+1364}}{22}$$

$$= 1.285 \quad \text{or} \quad -2.194.$$

From a table of trigonometric functions in radian measure one finds that

$$\theta/2 = 0.910 \quad \text{or} \quad -1.143,$$

and that

$$\theta = 1.820 \quad \text{or} \quad -2.286 \text{ radians}.$$

The main advantage of this method of solving the given equation lies in the fact that there is no difficulty in determining the quadrant in which an angle may lie. It is easy to bear in mind that negative values of t correspond to negative angles and positive values of t correspond to positive angles.

EVALUATION OF A TRIGONOMETRIC INTEGRAL

J. P. HOYT, United States Naval Academy

The purpose of this note is to present a quick and easy method for evaluating $\int \sin^{2m} x \cos^{2n} x dx$, where m and n are positive integers. No table of formulas is required.

To illustrate the method consider the integration of $\int \sin^4 x \cos^6 x dx$. Since $\sin nx = (1/2i)(e^{inx} - e^{-inx})$ and $\cos nx = \frac{1}{2}(e^{inx} + e^{-inx})$, we let $y = e^{ix}$, and find that $\sin nx = (1/2i)(y^n - 1/y^n)$ and $\cos nx = \frac{1}{2}(y^n + 1/y^n)$. Then

$$\sin^4 x = \frac{\left(y - \dfrac{1}{y}\right)^4}{2^4} = \frac{y^4 - 4y^2 + 6 - \dfrac{4}{y^2} + \dfrac{1}{y^4}}{2^4}.$$

If we notice that the powers of y decrease by 2 from term to term, we may write this expression in the following symbolic form:

$$\sin^4 x = \frac{1 - 4 + 6 - 4 + 1}{2^4}.$$

Now instead of expanding $\cos^6 x$ in the same manner and then forming the product, it is easier and quicker to multiply the expansion of $\sin^4 x$ by the expression for $\cos x$ six times in succession. For the coefficients of each expansion are obtained from the coefficients of the preceding one in the same manner as the coefficients of $(a+b)^{n+1}$ are obtained from the coefficients of $(a+b)^n$; that is, in the form used in Pascal's Triangle.

The successive expansions appear as follows in the symbolic notation:

$$\begin{aligned}
\sin^4 x &= (1 - 4 + 6 - 4 + 1)/2^4 \\
\sin^4 x \cos x &= (1 - 3 + 2 + 2 - 3 + 1)/2^5 \\
\sin^4 x \cos^2 x &= (1 - 2 - 1 + 4 - 1 - 2 + 1)/2^6 \\
\sin^4 x \cos^3 x &= (1 - 1 - 3 + 3 + 3 - 3 - 1 + 1)/2^7 \\
\sin^4 x \cos^4 x &= (1 + 0 - 4 + 0 + 6 + 0 - 4 + 0 + 1)/2^8 \\
\sin^4 x \cos^5 x &= (1 + 1 - 4 - 4 + 6 + 6 - 4 - 4 + 1 + 1)/2^9 \\
\sin^4 x \cos^6 x &= (1 + 2 - 3 - 8 + 2 + 12 + 2 - 8 - 3 + 2 + 1)/2^{10}
\end{aligned}$$

or

$$\sin^4 x \cos^6 x = \left(y^{10} + 2y^8 - 3y^6 - 8y^4 + 2y^2 + 12 + \frac{2}{y^2} - \frac{8}{y^4} - \frac{3}{y^6} + \frac{2}{y^8} + \frac{1}{y^{10}}\right) \Big/ 2^{10}.$$

Regrouping the last expression we obtain:

$$\sin^4 x \cos^6 x = \left[\left(y^{10} + \frac{1}{y^{10}}\right) + 2\left(y^8 + \frac{1}{y^8}\right) - 3\left(y^6 + \frac{1}{y^6}\right) - 8\left(y^4 + \frac{1}{y^4}\right) + 2\left(y^2 + \frac{1}{y^2}\right) + 12\right] \Big/ 2^{10}$$

which can be rewritten:

$$\sin^4 x \cos^6 x = (\cos 10x + 2 \cos 8x - 3 \cos 6x - 8 \cos 4x + 2 \cos 2x + 6)/2^9.$$

So immediately we have the final result:

$$\int \sin^4 x \cos^6 x dx = \left[\frac{\sin 10x}{10} + \frac{\sin 8x}{4} - \frac{\sin 6x}{2} - 2 \sin 4x + \sin 2x + 6x\right] \Big/ 2^9 + C.$$