Mean Temperature Difference and Heat Transfer Coefficient in Liquid Heat Exchangers

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The rate of heat flow between two fluids in a heat exchanger has customarily been expressed as being equal to the product of the area of the exchanger, the temperature difference between the two fluids averaged with respect to the length of area of the exchanger, and a coefficient of heat transfer. Where the coefficient of heat transfer can be assumed as constant throughout the apparatus, it was early demonstrated that the logarithmic mean of the temperature differences between the hot and cold liquids at the two ends of the apparatus is the correct average to be used.

For liquids, experimental data have more recently shown that the coefficient of heat transfer varies with temperature and thus with length of the exchanger, so that the logarithmic mean of the terminal temperature differences is not the correct average value. To make a correct design for such a case, it has been necessary to apply a graphical solution using values of temperature difference and coefficient of heat transfer occurring at small temperature intervals. To avoid this laborious procedure, it has been common practice to accept the error (of variable magnitude) incurred by employing a logarithmic mean temperature difference and some arbitrarily chosen average value of the coefficient of heat transfer. Since the over-all heat transfer coefficient in most liquid heat exchangers is found to approximate a linear relationship with the temperature of either heat transfer medium, it was thought desirable to make an integration for the general case for the purpose of determining what values of heat transfer coefficients and temperature difference should be generally used. A derivation has been carried out with the assumption that the heat transfer coefficient is a linear function of temperature for the cases where both heat transfer mediums are exchanging sensible heat, or where one medium is exchanging sensible heat and the other is at a constant temperature, such as a condensing or boiling fluid. This derivation is given at the close of the article. The resulting equation is:

\[ Q/A = \frac{U_1 \Delta T - U_2 \Delta T}{2.3 \log \frac{U_1 \Delta T}{U_2 \Delta T}} \] (1)

Thus when the heat transfer coefficient is a linear function of temperature, the rate of heat flow divided by the area, \( Q/A \), is equal to the logarithmic mean of \( U_2 \Delta T \) and \( U_1 \Delta T \), where \( U \) and \( \Delta T \) are the over-all heat transfer coefficient and temperature difference, respectively, at the ends of the apparatus. This method is a new departure in the design of heat transfer apparatus in that it uses a logarithmic mean of a combined function of heat transfer coefficient and temperature difference rather than a logarithmic mean of the temperature difference and some assumed function of the heat transfer coefficient.

After developing the above method of design, it was felt desirable to find some procedure of determining a particular value of the heat transfer coefficient which could be used with the logarithmic mean of the terminal temperature differences to give also the same result as Equation 1. A specific temperature was derived at which to compute the heat transfer coefficient to be used with the logarithmic mean of the terminal temperature differences. This derivation is given at the end of the paper. The resulting formula is too complicated to be easily used and so has been represented by a family of curves given by Figure 1. The ordinates of a family of curves is also indicated by this family of curves. It is necessary only to use the parameter:

\[ C = \frac{U_2 - U_1}{U_1} \]

For interpolation of experimental data where the terminal values of heat transfer coefficient are not known, it is necessary to use the parameter:

\[ C = \frac{t_2 - t_1}{1/b + t_1} \]

where \( b = \text{temp. coeff. of heat transfer coefficient in formula} \)

\[ U = a (1 + b t) \]

Thus, in interpreting experimental data, the observed value of heat transfer coefficient which has been calculated by use of a logarithmic mean temperature difference should be correlated with the properties of the fluid at the temperature indicated by this family of curves. It is necessary only to estimate the temperature coefficient of the heat transfer coefficient to do this. The variation of the factor, \( F \), with the parameter, \( C \), is not very great, so that even a rough value of \( C \) will permit a fairly accurate determination of the correct temperature.

The new method of designing heat exchangers is based on the assumption that the coefficient of heat transfer is a linear function of the temperature of either fluid and, since
the jacket, or 23,100 pounds per hour, resulting in an outlet water temperature of

An attempt was also made to derive the correct relationship for the case where the heat transfer coefficient is a parabolic function of temperature—i.e., \( U = at^2 \)—but a differential equation was obtained which could not be integrated for the general case.

**Examples**

A. **WHERE \( U \) IS PRACTICALLY A LINEAR FUNCTION OF \( t \).**

An exchanger is to be designed for 21,000 pounds of aniline per hour which is to be cooled from 125° to 25° C., with clean cooling water at 20° C. To give an optimum aniline velocity of around 4 feet per second, a convenient apparatus is found to be a 3-inch standard pipe jacketed with a 3-inch standard pipe. A convenient water rate is 5 feet per second through the jacket, or 23,100 pounds per hour, resulting in an outlet water temperature of 65° C. The length of the exchanger will be determined by the required heat transfer area. Values of the heat transfer coefficients of the aniline and of the water at the two ends of the apparatus are calculated by Equations 3 and 4. A film coefficient of 1000 is to be used to allow for the thermal resistances of the metal pipe and any deposit collecting thereon. The water film coefficients were adjusted to the basis of the inside diameter of the pipe and then over-all heat transfer coefficients were determined from these two film coefficients and the assumed pipe resistance. Heat transfer coefficients were calculated also at various temperatures suggested by previous methods of determining an average heat transfer coefficient. A graphical integration was finally made from values of heat transfer coefficients and temperature differences at various temperature intervals.

The graphical integration is made by Equation 10, for which values of \( 1/UAt \) are calculated at regular intervals of \( t \). Then the area under a curve of \( 1/UAt \) vs. \( t \) between the ordinates \( t_1 \) and \( t_2 \) is equal to \( \Delta t/At \). Values of \( \Delta t \) at any values of \( t \) are calculated directly from a heat balance between the two streams, as indicated by Equation 6. Values of \( U \) are determined from values of the film coefficients calculated at each temperature interval. The film coefficient for the cooling aniline is obtained from the Dittus and Boelter type of equation forcooling liquids:

\[
h = 0.020 \frac{d}{d} \left( \frac{c}{c} \right)^{0.8} \left( \frac{t}{t} \right)^{0.8} \left( \frac{d}{d} \right)^{0.8}
\]

where \( h \) = film coefficient

\( k \) = thermal conductivity

\( d \) = inside pipe diameter

\( c \) = sp. heat

\( \mu \) = viscosity

and \( G \) = mass velocity (all in self-consistent units)

Values of most of the variables are nearly constant through the apparatus. Thus \( k = 0.103 \) pound-Centigrade unit per hour per square foot per \( \deg \) C./foot at 125° C.; and 0.108 at 25° C.; \( c = 0.55 \) at 125° C., and 0.5 at 25° C.; \( d = (0.207/12) \) foot; and \( G = (250 \times 3600) \) pounds per hour per square foot. Thus \( k \) and \( c \) vary in different ways with \( t \), so that \( (k/\mu, c/\mu) \) in Equation 2 is essentially independent of temperature. For this case Equation 2 becomes:

\[
h = \frac{581}{\sqrt{\mu}}
\]

where \( \mu = \text{pounds/hour/foot} \) (\( \mu = 2.42 z \), where \( z \) is in centipoises)

Values of the viscosity of aniline and calculated aniline film coefficients, \( h_a \), are given in Table I.

### Table I. DATA ON VISCOSITY OF ANILINE AND CALCULATED ANILINE FILM COEFFICIENTS

<table>
<thead>
<tr>
<th>( t_c )</th>
<th>( t_w )</th>
<th>( t_d )</th>
<th>( h_a )</th>
<th>( h_w )</th>
<th>( U )</th>
<th>( \Delta t / (1 / UAt) \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>65</td>
<td>0.44</td>
<td>1.03</td>
<td>565</td>
<td>1960</td>
<td>305</td>
</tr>
<tr>
<td>120</td>
<td>62.8</td>
<td>0.48</td>
<td>1.09</td>
<td>533</td>
<td>1930</td>
<td>295</td>
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<tr>
<td>115</td>
<td>55.3</td>
<td>0.50</td>
<td>1.13</td>
<td>490</td>
<td>1870</td>
<td>280</td>
</tr>
<tr>
<td>110</td>
<td>55.3</td>
<td>0.72</td>
<td>1.32</td>
<td>441</td>
<td>1800</td>
<td>262</td>
</tr>
<tr>
<td>105</td>
<td>44.8</td>
<td>0.87</td>
<td>1.45</td>
<td>401</td>
<td>1740</td>
<td>246</td>
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<tr>
<td>100</td>
<td>40.3</td>
<td>1.1</td>
<td>1.63</td>
<td>357</td>
<td>1680</td>
<td>228</td>
</tr>
<tr>
<td>95</td>
<td>35.8</td>
<td>1.7</td>
<td>2.03</td>
<td>327</td>
<td>1620</td>
<td>212</td>
</tr>
<tr>
<td>90</td>
<td>31.3</td>
<td>2.13</td>
<td>2.44</td>
<td>297</td>
<td>1560</td>
<td>195</td>
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<tr>
<td>85</td>
<td>26.8</td>
<td>2.72</td>
<td>2.85</td>
<td>272</td>
<td>1500</td>
<td>178</td>
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<tr>
<td>80</td>
<td>22.3</td>
<td>3.52</td>
<td>3.46</td>
<td>248</td>
<td>1440</td>
<td>164</td>
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<td>75</td>
<td>17.9</td>
<td>4.50</td>
<td>3.99</td>
<td>222</td>
<td>1380</td>
<td>149</td>
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<tr>
<td>70</td>
<td>13.5</td>
<td>5.59</td>
<td>4.59</td>
<td>200</td>
<td>1320</td>
<td>134</td>
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</tbody>
</table>

Water film coefficients are calculated by the equation:

\[
h_w = 240 \left( 1 + 0.0127t \right) u^{0.4} D^{0.2}
\]

where \( t \) = water temp., \( \circ \) C.

\( u \) = water velocity, feet per sec.

\( D \) = equivalent diameter, or clearance, inches

Since \( u = 5 \) and \( D = 0.699 \), \( h_w = 935 \left( 1 + 0.0127t \right) \), based on outside diameter of pipe. Then based on inside diameter, \( h_w = 1073 \left( 1 + 0.0127t \right) \), values of which are included in Table I. Including a coefficient of 1000 to take care of the pipe and scale resistances (for clean liquids), over-all heat transfer coefficients are calculated at each temperature and
included in Table I. Values of $1/U\Delta t$ were computed for each temperature and plotted on Figure 2. The area under the curve between $t_1 = 25^\circ$ and $t_2 = 125^\circ$ C. is found to be $0.025 = A/(\text{sec})$. Since $w$ is 21,000 pounds per hour, and $c$, the specific heat, is 0.525, $A$ is $0.025 \times 21,000 \times 0.525 = 275$ square feet of cooling surface, obtained by the graphical integration, which is the correct value of area for this problem.

Figure 2. Example A. Cooling Aniline with Water

$U =$ heat transfer coefficient

$\Delta t =$ temperature of aniline, $^\circ$ C.

$\Delta t =$ temperature difference between aniline and water

By method 1, $(U\Delta t)_{\text{log}} =$ logarithmic mean of $U_1 \Delta t_1$ and $U_2 \Delta t_2$, or of $141 \times 60$ and $305 \times 5$ or 4050. Since $Q = 21,000 \times 0.525 (125 - 25) = 1,103,000$ P. C. U. per hour, $A = 1,103,000/4050 = 275$ square feet.

By method 2, $A\Delta t = \frac{1}{w} = 0.0833$. $C = (U_2 - U_1)/U_1 = (305 - 141)/141 = 1.16$. Then from Figure 1, $F = 0.255$, $(t_2 - t_1)/(t_2 - t_1) = (t_2 - 25)/(125 - 25) = 0.253; t_1 = 50.3^\circ$ C.; and $U_2 = 180$. The logarithmic mean $\Delta t$ of 5 and 60 = 22.2, so $U_2 \Delta t_{\text{log}} = 180 \times 22.2 = 4000$. Then $A = 276$ square feet.

Figure 2 shows a plot of the calculated over-all heat transfer coefficient against temperature of the aniline and indicates how nearly $U$ varies as a straight-line relationship with $t$. A summary of the resulting calculations is given by Table III.

The two recommended methods give results on both sides of the correct graphical integration, with method 2 being closer. Either method, however, gives results checking the correct cooling surface within one per cent. Any of the previously applied approximation methods would have been considerably on the unsafe side.

B. WHERE $U$ IS NOT A LINEAR FUNCTION OF $t$. Straw oil is to be heated from 80° to 200° F., flowing at an average velocity of 3 feet per second through a standard 2-inch horizontal pipe using steam at 227° F. in a jacket outside the pipe. The oil has a mean specific heat of 0.47, a mean specific gravity of 0.85, and a mean thermal conductivity of 0.078 in English units. The viscosities of the oil are 18 and 4 centipoises at 80° and 200° F., respectively, and a plot of viscosity vs. temperature on logarithmic paper may be considered a straight line. The outside and inside diameters of the pipe are 2.38 and 2.07 inches, respectively; the thermal conductivity of the pipe is 35 in English units. Since the oil side resistance is controlling, the heat transfer coefficient on the steam side will be assumed constant at 2150.

In this case the heat transfer coefficient does not vary as the 0.8 power of Reynolds number, since the Reynolds number is near the critical value. Values of the oil film coefficient, $h_n$, are obtained from a plot of $\frac{h_d}{k} = (\frac{c}{k})^4$ vs. $\frac{dG}{T}$ given by Figure 3.

The equation for the over-all coefficient is:

$$1 \frac{1}{U} = 2150 (2.38/2.07) + 0.154/12 + \frac{1}{h_n}$$

$$= 0.000751 + \frac{1}{h_n} \quad (5)$$

Table II gives calculated values of over-all coefficient, temperature difference, and of $1/U\Delta t$ at various temperature intervals. Figure 4 shows a plot of $U$ vs. $t$ and of $1/U\Delta t$ vs. $t$. The area under the latter curve from $t = 80^\circ$ to $t = 200^\circ$ F. is found to be 0.0321 = $A/(\text{sec})$. Since $w$ is 13,350 pounds per hour and $c$ is 0.47, $A = 0.0321 \times 13,350 \times 0.47 = 202$ square feet, which is the correct solution for this problem.

By method 1, $(U\Delta t)_{\text{mean}} =$ logarithmic mean of $U_1 \Delta t_1$ and $U_2 \Delta t_2$, or of $21 \times 27$ and 60 = 3340. Since $Q/A = 13,350 \times (200 - 80) \times 0.47 = 753,000$ B. t. u. per hour, $A = 753,000/3340 = 225$ square feet.

By method 2, $\Delta t/\Delta t = 147/27 = 5.43$. $C = (U_2 - U_1)/U_1 = 2.27$. From Figure 1, $F = 0.54$; then $t_4 = 145^\circ$ F., $U_4 = 54.5$. The logarithmic mean $\Delta t$ of 27 and 147, or 70.7. $U_2 \Delta t_{\text{mean}} = 3850$. $A = 196$ square feet.

Table II. Calculated Values at Various Temperature Intervals

<table>
<thead>
<tr>
<th>$t$</th>
<th>$DG/k$</th>
<th>$G/k$</th>
<th>$AD/k$</th>
<th>$h$</th>
<th>$1/A$</th>
<th>$1/U$</th>
<th>$T = 1$</th>
<th>$U$</th>
</tr>
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<tbody>
<tr>
<td>80</td>
<td>18.0</td>
<td>2960</td>
<td>262</td>
<td>47.5</td>
<td>21.5</td>
<td>0.0455</td>
<td>0.0473</td>
<td>147</td>
</tr>
<tr>
<td>90</td>
<td>15.0</td>
<td>2720</td>
<td>219</td>
<td>65.6</td>
<td>26.7</td>
<td>0.0347</td>
<td>0.0348</td>
<td>147</td>
</tr>
<tr>
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<td>57.0</td>
<td>0.0270</td>
<td>0.0276</td>
<td>127</td>
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<td>3770</td>
<td>156</td>
<td>92.4</td>
<td>42.1</td>
<td>0.0237</td>
<td>0.0248</td>
<td>117</td>
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<td>0.0219</td>
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<td>165.8</td>
<td>74.8</td>
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<td>0.0167</td>
<td>0.37</td>
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<td>170</td>
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<td>0.0152</td>
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<td>0.57</td>
</tr>
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<td>70.1</td>
<td>150.5</td>
<td>68.6</td>
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<tr>
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<td>0.0142</td>
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</tr>
<tr>
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<td>141.1</td>
<td>73.0</td>
<td>0.0145</td>
<td>0.0145</td>
<td>0.27</td>
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</table>

Figure 3 shows a curve of heat transfer coefficient vs. temperature which is considerably concave downward. A comparison was again made of area determined by the use of various methods as shown by Table III. For this case, method 1 recommended herein gives an area too large, and method 2 gives one slightly too small. The reason for this difference can be seen by examination of Figure 4. The first method is based on the assumption that $U$ follows the dashed straight line $A$. Method 2 assumes that $U$ is expressed by the dashed straight line $B$ which is a closer approximation to the true curve. The true curve will practically always lie between $A$ and $B$ so that the true area will be between the areas computed by the two methods given herein when $U$ is not a linear function of $t$. 

Figure 3. Heat Transfer Coefficients for Heating Heavy and Medium Oils
Correct method: Graphical integration 277 202
Recommended methods: Method 1 273 - 0.7 225 11.4
Method 2 276 0.4 196 -12.0
Methods previously used: \( U_{av} = \frac{\Delta T_{lm}}{\Delta t} \)
227 - 17.5 202 0
\( U_{av} = \frac{\Delta T_{lm}}{\Delta t} \)
233 - 18.9 240 16.8
\( U_{av} = \frac{\Delta T_{lm}}{\Delta t} \)
192 -49.7 192 -5.0
\( A (t = \Delta T_{lm}) \) and \( \Delta t \)
191 - 9.4
Substituting, \( a(1 + b_t) (T_4 - t_1) - a(1 + b_t) (T_4 - t_2) \)
\( \left( \ln \frac{T_1 - t_1}{T_4 - t_1} - \ln \frac{1 + b_t}{1 + b_t} \right) = A \)
(14)
Simplifying by use of Equation 7 and rearranging:
\( t_1 = t_2 \)
\( a(1 + b_t) (T_4 - t_1) - a(1 + b_t) (T_4 - t_2) \)
\( \left( \ln \frac{T_1 - t_1}{T_4 - t_1} - \ln \frac{a(1 + b_t)}{a(1 + b_t)} \right) = A \)
(15)
Substituting, \( U_1 = a(1 + b_t); U_2 = a(1 + b_t) \)
\( \Delta t_1 = T_1 - t_1; \) and \( \Delta t_2 = T_2 - t_2 \)
Combining Equations 16 and 6 gives:
\( Q/A = \frac{U_1 \Delta t_1 - U_2 \Delta t_2}{2.3 \log U_1 \Delta t_1} \)
(17)
SOLUTION OF TEMPERATURE TO BE USED IN DETERMINING A PARTICULAR VALUE OF HEAT TRANSFER COEFFICIENT WHICH, WHEN USED WITH A LOGARITHMIC MEAN TEMPERATURE DIFFERENCE, GIVES SAME RESULT AS EQUATION 1.
An instantaneous value of the heat transfer coefficient, \( U_t \), can be found so that:
\( Q = U_t \left( \frac{\Delta t_1 - \Delta t_2}{2.3 \log \frac{\Delta t_1}{\Delta t_2}} \right) \)
(17)
Substituting,
\( U_t = a(1 + b_t); U_1 = a(1 + b_t); U_2 = a(1 + b_t) \)
Equation 17 becomes:
\( a(1 + b_t) \Delta t_1 = a(1 + b_t) \Delta t_2 \)
\( a(1 + b_t) = \frac{2.3 \log \frac{(1 + b_t) \Delta t_1}{(1 + b_t) \Delta t_2}}{2.3 \log \frac{\Delta t_1}{\Delta t_2}} \)
(18)
Let \( F = \frac{t_2 - t_1}{t_1 - t_2} \)
\( F \) is thus a fraction, which, multiplied by the total temperature rise of one fluid stream and added to the lower terminal temperature of the same fluid stream, gives the temperature at which to calculate the \( U \) to be used with a logarithmic mean \( \Delta t \).

Figure 5. Typical Temperature Distribution

A family of curves for solving this equation is given by Figure 1.

NOMENCLATURE (REFER TO FIGURE 5)
\( T \) = temp. at any point of cold fluid, °C.
\( T_t \) = temp. at any point of hot fluid, °C.
\( t_1, t_2 \) = terminal temp. of cold fluid, °C.
\( T_1, T_2 \) = terminal temp. of hot fluid, °C.
\( U_t \) = heat transfer coefficient at any point, P. C. U./hour/sq. ft./°C.
\( A \) = heat transfer area, sq. ft.
\( Q \) = heat transfer rate, P. C. U./hour.
\( w \) = rate of flow of cold liquid, lb./hour.
\( W \) = rate of flow of hot liquid, lb./hour.
of W. H. McAdams and to thank him for permission to use

The writer wishes to acknowledge the valuable criticisms of W. H. McAdams and to thank him for permission to use the material in example B which was taken from his unpublished notes.


Improved Process for Physical Development of Plates, Films, and Lantern Slides


The introduction of the physical development of negatives, which is an adaptation of the method used to develop the colloidion emulsion plates of the middle of the last century, has been ascribed by various authors to Sterry (16), Neuhaus (12), and Kogelmann (6), but the later developments in the method are to be found largely in the publications of the Lumière and Seyewetz (9) and of Lupp-Cramer (10-12). Wall (17) claims that the method applied to dry plates was first suggested by Abney in 1878.

ADVANTAGES OF PHYSICAL DEVELOPMENT

Physical development, in the sense the term is used today, is to be distinguished from ordinary methods which are generally employed in all photographic work, sometimes called "chemical development" in order to introduce a difference in terminology, by the fact that the developing bath contains a silver salt in solution from which colloidal silver is obtained by reduction with a suitable agent, also in the solution, and the production of the negative image results from the deposition of the nascent silver upon the silver nuclei forming or originating in the latent image. Both methods are, of course, chemical, and the developing baths resemble each other by the further fact that they both contain the same developers as reducing agents. Physical development occurs only when the bath contains one of the usual photographic developers, which must function first in some manner by preparing the latent image to receive the silver deposit (6, 7, 12, 15).

Physical development has been little used outside the laboratory, where it has proved a valuable adjunct in research upon emulsions and upon the theory of image formation, and information upon the method, while widely distributed through the literature, is rarely found outside of very restricted fields. It has so many advantages, however, that it should long ago have been put upon a practical operating basis and introduced as a part of laboratory technic wherever photographic requirements are necessary.

The reasons why this has not been done are readily found in the difficulties that surround the present methods. Long exposures, up to ten to twenty times normal, are required. The development takes from 6 to 48 hours with ordinary negative emulsions. The bath must be freshly renewed every hour or two. The solutions, usually strongly acid, attack the emulsion, and the most careful handling is necessary to avoid marring the surface. It requires all one's personal attention to finish one negative at a time. Finally, the precipitating silver makes the operation unseemingly dirty.

Complete descriptions of the methods currently in use may be found in the literature already cited, and in a few available textbooks (1-4, 18). In general, these methods are all similar and, aside from those of Sterry (16) and of Neuhaus (18) which consist of a bath of hypo, ammonium sulfocyanate, sodium sulfite, silver nitrate, and a developer-reducer, and from one proposed by the Lumière and Seyewetz (8) of a neutral solution of silver nitrate and sodium sulfite, all are made up of baths containing citric acid in high concentration, metol, and silver nitrate, sometimes with the addition of sodium citrate, sodium sulfite, gum arabic, etc.

Most authors have noted that the image from physical development is composed of an extremely fine grain, and that the gradation is beautifully exact. The extreme fineness of the grain size, coupled with the fact that this grain is formed independently of the grain of the original emulsion, constitutes the chief advantage of the method. One may thus use coarse-grain superspeed emulsions to obtain fine-grain images. This factor becomes of great importance where enlargements of unusual size are to be made from small negatives.

Aside from the many beautiful and useful aspects which physically developed negatives have from an artistic standpoint (12), such negatives and prints, or enlargements made from them, should have a definite appeal to the scientist and laboratory worker in nearly any field where photography is applicable.

Wherever reproduction as close as possible to the original and with an absence of unnatural contrasts is desired, for intricate detail, and for clean and extraordinarily sharp line work particularly for measurements, this type is better adapted than the ordinarily used methods of photographic development. It should be of value in microphotography and cross-section work especially, in biological work (particularly photographs from life), and in x-ray work where grain is a problem and undue contrasts are not desirable; it should prove especially valuable in spectrum photography where both sharpness of lines and correct comparative densities are important.

SIMPLIFIED PROCESS FOR PHYSICAL DEVELOPMENT

A study undertaken to improve existing methods of physical development showed, after repeated experiments with the

\[ c = \text{sp. heat of cold liquid, P. C. U./lb./}^\circ \text{C.} \]

\[ C = \text{sp. heat of hot liquid, P. C. U./lb./}^\circ \text{C.} \]